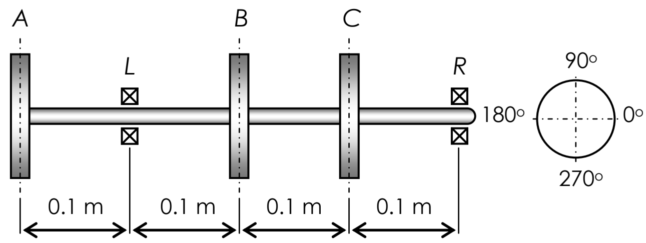
QUESTION 1

The figure shows a completely balanced system comprising three 20 kg mass discs *A*, *B* and *C*. Some modifications have been done to discs *A* and *C*. At disc *A*, a mass of 0.4 kg is added at radius of 0.4 m and direction 90°. At disc *C*, a mass of 0.2 kg is removed at radius of 0.5 m and direction 180°. Determine

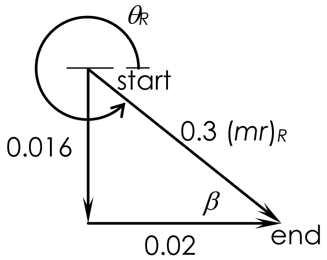


a. the dynamic force acting on bearings X and Y if the shaft is rotating at 3000 rpm.

b. the magnitude and direction of masses to be added at discs *B* and *C* each at radius 0.2 m in order to balance the system.

SOLUTION

Plane	m	r	mr	d	mrd	θ
A	0.4	0.4	0.16	– 0.1	– 0.016	90°
L			$(mr)_L$	0	0	θ_L
C	– 0.2	0.5	– 0.1	0.2	– 0.02	180°
R			$(mr)_R$	0.3	0.3 $(mr)_R$	θ_R



$0.3\ (mr)_R = \sqrt{0.016^2 + 0.02^2} = 0.0256$

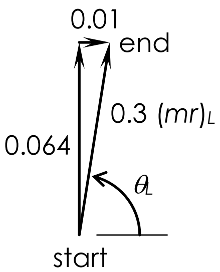
$(mr)_R = 0.0854$ kg·m

$\beta = \tan^{-1}(0.016/0.\ 02) = 38.66°$

$\theta_R = 360° – 38.66° = 321.34°$

$F_R = (mr)_R\ \omega^2 = 0.0854\ [2\pi(3000)/60]^2 = 8428.6$ N

Plane	m	r	mr	d	mrd	θ
A	0.4	0.4	0.16	0.4	0.064	90°
L			$(mr)_L$	0.3	0.3 $(mr)_L$	θ_L
C	– 0.2	0.5	– 0.1	0.1	– 0.01	180°
R			$(mr)_R$	0	0	θ_R



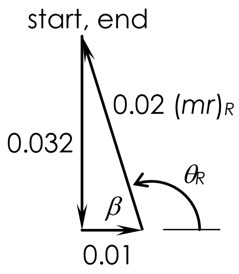
$0.3\ (mr)_R = \sqrt{0.064^2 + 0.01^2} = 0.0216$

$(mr)_R = 0.0854$ kg·m

$\theta_R = \tan^{-1}(0.064/0.\ 01) = 81.1°$

$F_R = (mr)_R\ \omega^2 = 0.216\ [2\pi(3000)/60]^2 = 21318$ N b.

Plane	m	r	mr	d	mrd	θ
A	0.4	0.4	0.16	– 0.2	– 0.032	90°
B	m_B	0.2	0.2 m_B	0	0	θ_L
C	– 0.2	0.5	– 0.1	0.1	– 0.01	180°
C	m_C	0.2	0.2 m_C	0.1	0.02 m_C	θ_R



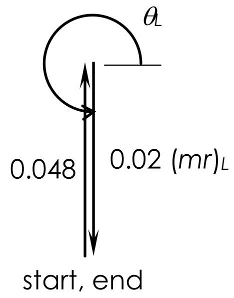
$$0.02\,(mr)_R = \sqrt{0.032^2 + 0.01^2}$$

$$(mr)_R = 1.68 \text{ kg·m}$$

$$\beta = \tan^{-1}(0.032/0.\ 01) = 72.65^\circ$$

$$\theta_R = 180^\circ - 72.65^\circ = 107.35^\circ$$

Plane	m	r	mr	d	mrd	θ
A	0.4	0.4	0.16	0.3	0.048	90°
B	m_B	0.2	0.2 m_B	0 1	0.02 m_B	θ_L
C	– 0.2	0.5	– 0.1	0	0	180°
C	m_C	0.2	0.2 m_C	0	0	θ_R



$$0.02\,(mr)_L = 0.048$$

$$(mr)_L = 2.4 \text{ kg·m}$$

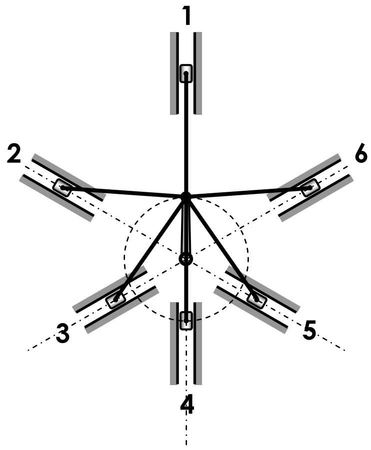
$$\theta_R = 270^\circ$$

QUESTION 2

A 6–cylinder engine has an equal dimension of crank radius *r*, connecting rod *L*, piston mass *m* and rotating at an angular speed ω. Investigate the balance condition if the engine is developed as

a. a 4–stroke in–line engine with firing order 162534. Distances between cylinders are equal.

b. a radial engine with the cylinder arrangement as shown.

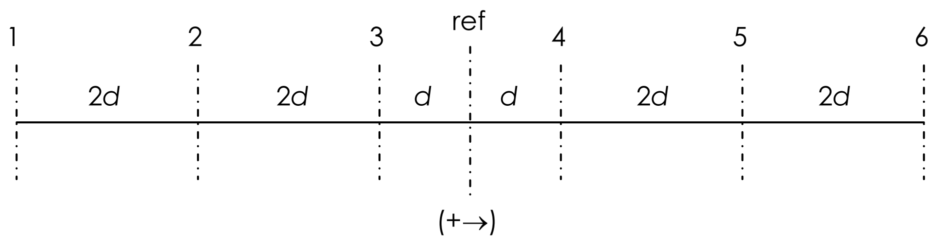


SOLUTION

a. For a 4 stroke 6 cylinder engine

Crank angle: $\theta = 720^\circ/6 = 120^\circ$

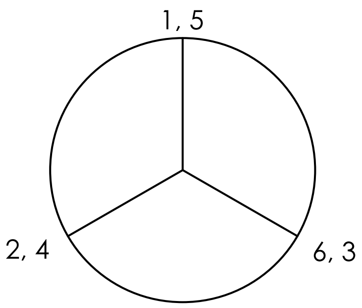
$2\theta = 240^\circ$



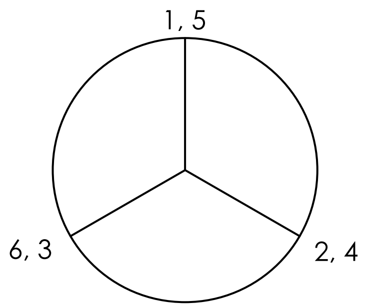
Plane	*m*	*r*	*mr*	*d*	*mrd*
1	*m*	*r*	*mr*	–5*d*	–5*mrd*
2	*m*	*r*	*mr*	–3*d*	–3*mrd*
3	*m*	*r*	*mr*	–*d*	–*mrd*
4	*m*	*r*	*mr*	*d*	*mrd*
5	*m*	*r*	*mr*	3*d*	3*mrd*
6	*m*	*r*	*mr*	5*d*	5*mrd*

For firing order 162534

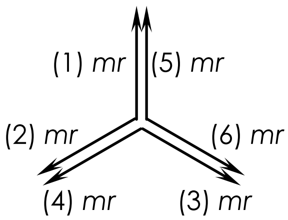
Primary crank



Secondary crank

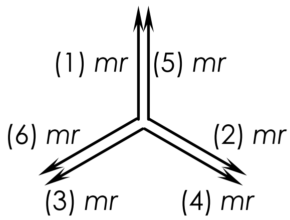


Primary Force (refer θ and *mr*)



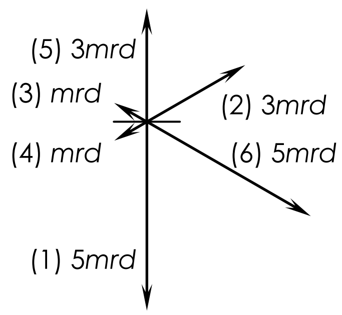
Primary Force balanced

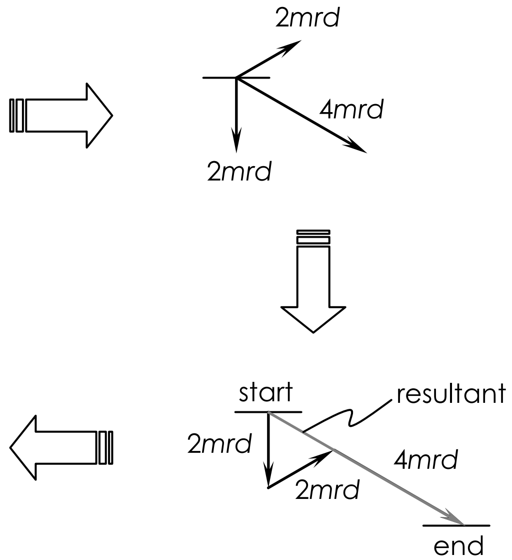
Secondary Force (refer to 2θ and *mr*)



Secondary Force balanced

Primary Couple (refer θ and *mrd*)



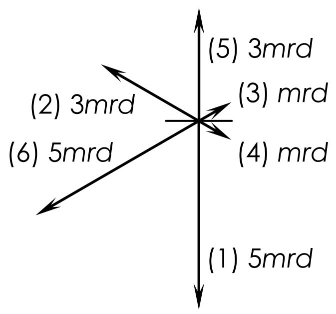


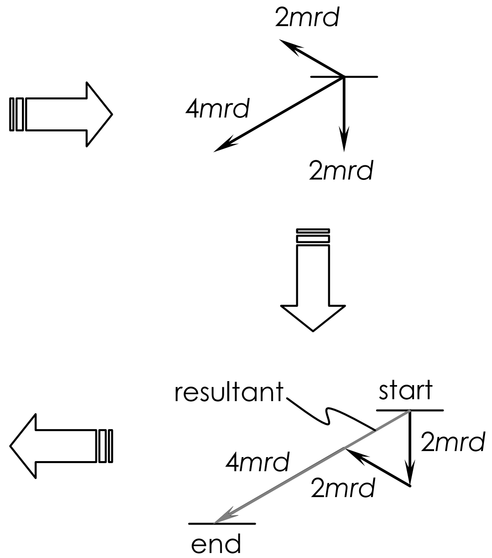
Primary Couple

Resultant Unbalance = 6 *mrd* 30°

∴ Primary Couple = 6 *mrd* ω^2 N·m 30°

Secondary Couple (refer 2θ and *mrd*)





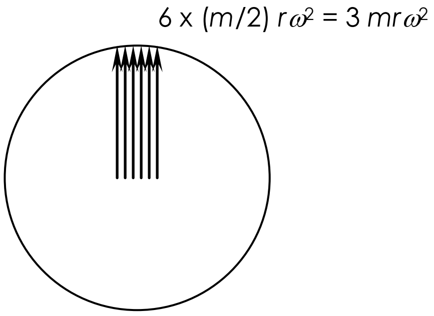
Secondary Couple

Resultant Unbalance = 6 *mrd* 30°

∴ Secondary Couple = 6 *mrd* ω^2 N·m 30°

b. Radial Engine

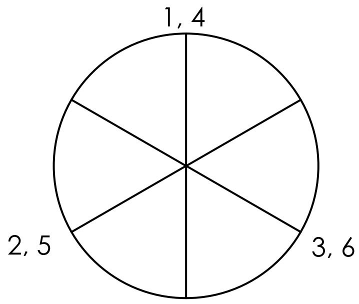
Primary Direct



Primary Force Unbalanced

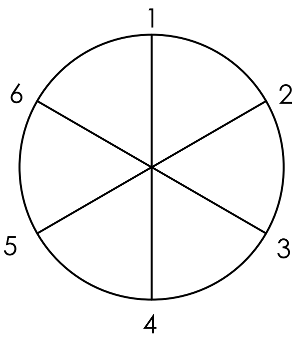
= = 3 $mr\omega^2$ N·m

Primary Reverse



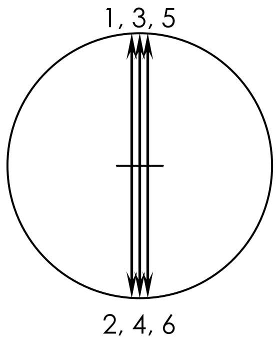
Primary Reverse Balanced

Secondary Direct



Secondary Direct Balanced

Secondary Reverse



Secondary Reverse Balanced

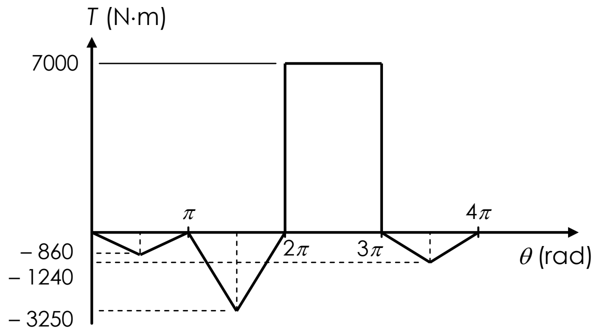
QUESTION 3

The figure shows a crank effort diagram for a 4 stroke engine. A constant torque is supplied to the load while the engine is running at a mean speed of 200 rpm. Determine

a. the mean torque and power of the engine.

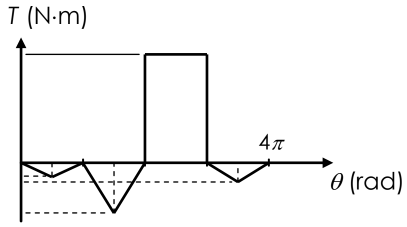
b. the maximum fluctuation in energy for 1 cycle.

c. if the mass moment of inertia of the flywheel is 1900 kg·m², find the maximum and minimum speed for 1 cycle.

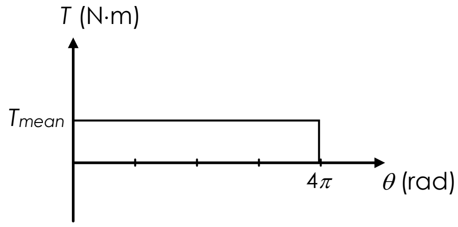


SOLUTION

Variable Input



Constant Output



Area LHS = Area RHS

$\frac{1}{2}(-860)(\pi) + \frac{1}{2}(-3250)(\pi) + 7000\ (\pi) + \frac{1}{2}(-1240)(\pi) = T_{mean}\ (4\pi)$

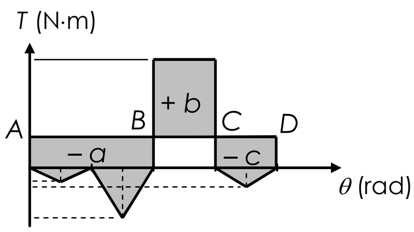
$(-430 - 1625 + 7000 - 620)\ (\pi) = T_{mean}\ (4\pi)$

$T_{mean} = 1081.25\ \pi$

a. Mean Torque = 1081.25 N·m

Engine Power = $T_{mean}\ \omega_{mean}$ = 1081.25 [2π (200)/60] = 22.645.6 kW

Super impose the two graphs



Area below T_{mean} = Area above T_{mean}

Area a + Area c = Area b

Area a = 1081.25 (2π) + 430π + 1625π = 4217.5π Area b = (7000 – 1081.25)4π = 5918.75π

Area c = 1081.25 π + 620π = 1701.25π

OK

Fluctuation of Energy in 1 cycle

Let the Energy at $A = U$

Energy at $B = U - a = U - 4217.5\pi$ minimum E

Energy at $C = U - a + b = U - 4217.5\pi + 5918.75\pi = U + 1701.25\pi$ maximum E

Energy at $C = U - a + b - c = U + 1701.25\pi - 1701.25\pi = U$

b. Maximum Fluctuation of Energy in 1 cycle $(\beta E) = (U + 1701.25\pi) - (U - 4217.5\pi)$

$= 5918.75\pi = 18{,}594.3$ N·m

c. Mass moment of Inertia of Flywheel, $I = \dfrac{\beta E}{\alpha \omega_{mean}^2}$

$$1900 = \frac{5918.25\pi}{\alpha\left(\frac{2\pi(200)}{60}\right)^2}$$

Coefficient of fluctuation of speed, $\alpha = 0.0223$

$N_{mean} = \dfrac{N_{\max} + N_{\min}}{2}$ $\qquad N_{max} + N_{min} = 400 \qquad (1)$

$\alpha = \dfrac{N_{\max} - N_{\min}}{N_{mean}}$ $\qquad N_{max} - N_{min} = 8.92 \qquad (2)$

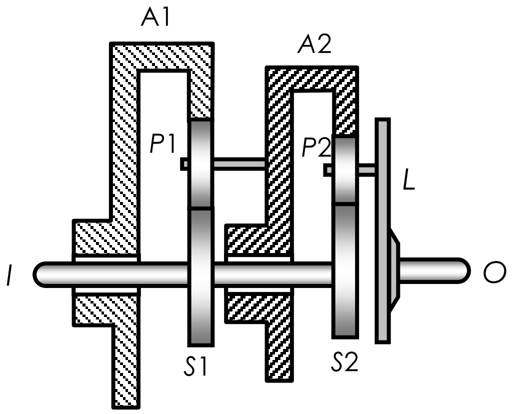
(1) + (2) $\quad 2 N_{max} = 408.92$

$N_{max} = 204.46$ rpm

(1) – (2) $\quad 2 N_{min} = 391.08$

$N_{min} = 195.54$ rpm

QUESTION 2



The figure shows a compound epicyclic gearset. The two gears *S*1 and *S*2 are integral with the input shaft *I*. The planet *P*1 revolves on a pin attached to the arm *L* which is integral with the output shaft *O*. The number of teeth are, t_{S1}= 22 ; t_{S2}= 31 ; t_{A1}= 88 ; t_{A2}= 93 and the gear efficiency is 90%. If the input power to the driving shaft is 22.5 kW at + 3000 rpm, calculate

a. the speed of shaft *O* (N_O) if gear *A*1 is rotating at – 2000 rpm.

b. The output torque (T_O), input torque (T_i) and braking torque (T_b) if gear *A*1 is fixed.